



Oxford Cambridge and RSA

Wednesday 18 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.



- 1 The table shows some values of x and the associated values of $f(x)$.

x	1.9	2.0	2.1
$f(x)$	0.21692	0.2	0.18484

(a) Determine an estimate of $f'(2.0)$ using the forward difference method. [2]

(b) Determine an estimate of $f'(2.0)$ using the central difference method. [2]

- 2 The numbers p and q are approximated by

$$P = 323 \text{ and } Q = 162.$$

P has been found by **rounding** p to the nearest whole number.

Q has been found by **chopping** q to the nearest whole number.

(a) (i) Find the maximum possible relative error in using P to approximate p . [1]

(ii) Find the maximum possible relative error in using Q to approximate q . [1]

(b) Determine the range of possible values of $R = \frac{200}{p-2q}$. [3]

(c) Explain why your answer to part (b) is so large. [1]

- 3 Approximations to $\int_{0.5}^{1.3} \sqrt{1+x^3} dx$ using the midpoint rule, the trapezium rule and Simpson's rule with $n = 1$ and $n = 2$ are shown in the table. The table is incomplete.

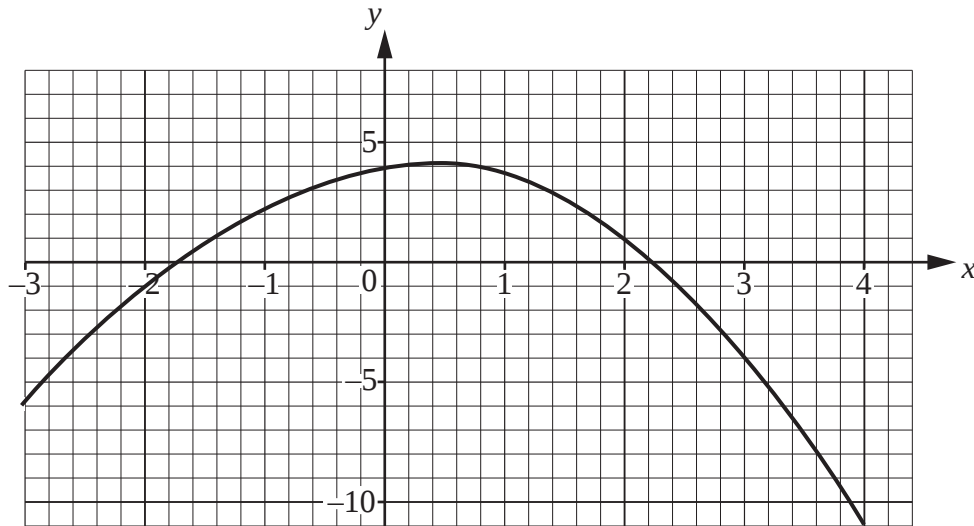
n	M_n	T_n	S_{2n}
1			1.081 111
2	1.074 256		

(a) Complete the copy of the table in the **Printed Answer Booklet**. Give your answers to 6 decimal places. [4]

(b) **Without doing any further calculations**, state the value of $\int_{0.5}^{1.3} \sqrt{1+x^3} dx$ as accurately as possible. You must justify the precision quoted. [1]

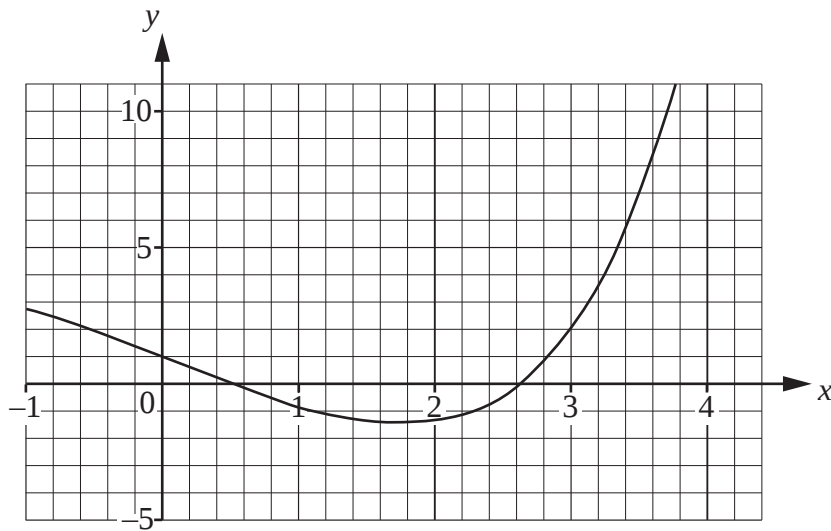
- 4 The Newton-Raphson method is to be used to find the positive root of the equation $\tanh x - x^2 + 4 = 0$.

The diagram shows part of the graph of $y = \tanh x - x^2 + 4$.



- (a) On the copy of the diagram in the **Printed Answer Booklet**, show how the Newton-Raphson method works to find x_1 using the starting value $x_0 = 1$. [1]
- (b) Use the Newton-Raphson method using the starting value $x_0 = 1$ to determine the values of x_1 and x_2 correct to **8** decimal places. [4]
- (c) Continue the iteration to determine the value of the positive root of the equation $\tanh x - x^2 + 4 = 0$ correct to **7** decimal places. [2]

- 5 The diagram shows part of the graph of $y = \sinh x - 3x + 1$.



The largest positive root, α , of the equation $\sinh x - 3x + 1 = 0$ is to be found using the secant method with $x_0 = 2$ and $x_1 = 3$.

Table 5.1 shows the associated spreadsheet output.

Table 5.1

	A	B	C	D	E
2	r	x_r	$f(x_r)$	x_{r+1}	$f(x_{r+1})$
3	0	2	-1.37314	3	2.01787
4	1	3	2.017875	2.404935	-0.7211
5	2	2.404935	-0.72109	2.561598	-0.2451
6	3	2.561598	-0.24513	2.642285	0.06018
7	4	2.642285	0.060175	2.626382	-0.0035
8	5	2.626382	-0.00348	2.627252	-5E-05
9	6	2.627252	-4.5E-05	2.627264	3.5E-08

- (a) Write down a suitable formula for cell D4. [2]
- (b) State the most accurate approximation to α in the spreadsheet output. [1]
- (c) Determine whether this approximation is accurate to 6 decimal places. [2]
- (d) Explain what would happen if the initial entry in cell D3 were to be changed from 3 to 1. [1]

Table 5.2 shows some further analysis of the sequence of approximations to α .

Table 5.2

A	B	C	D	E
r	x_r	$x_{r+1} - x_r$	$\frac{x_{r+2} - x_{r+1}}{x_{r+1} - x_r}$	$\frac{x_{r+2} - x_{r+1}}{(x_{r+1} - x_r)^2}$
1	3	-0.5951	-0.2633	0.44242
2	2.404935	0.15666	0.51504	3.28756
3	2.561598	0.08069	-0.1971	-2.4427
4	2.642285	-0.0159	-0.0547	3.44193
5	2.626382	0.00087	0.01315	15.1113
6	2.627252	1.1E-05		
7	2.627264			

(e) Explain what may be inferred from the entries in column D about the order of convergence of this sequence of approximations to α . [2]

(f) Explain what may be inferred from the entries in column E about the order of convergence of this sequence of approximations to α . [2]

6 The midpoint rule is used to find a sequence of approximations to $\int_0^1 \sqrt{\sin x} dx$.

The associated spreadsheet output, together with some further analysis, is shown in the table.

n	M_n	difference	ratio
1	0.6924056		
2	0.6615057	-0.0308999	
4	0.6498274	-0.0116782	0.37794
8	0.6454773	-0.0043501	0.3725
16	0.6438811	-0.0015962	0.36692
32	0.643302	-0.0005791	0.36281
64	0.6430936	-0.0002085	0.35996
128	0.6430189	-7.463E-05	0.35801

(a) Explain what the entries in the ratio column tell you about the order of the midpoint rule in this case. [2]

(b) Use the information in the table to determine the most accurate estimate of $\int_0^1 \sqrt{\sin x} dx$ possible. You must justify the precision quoted. [5]

- 7 The equation $8 \ln(x+2) - 3x + 1 = 0$ has a positive root, γ , and a negative root, δ .

It is proposed that the iterative formula

$$x_{n+1} = g(x_n), \text{ where } g(x) = 8 \ln(x+2) - 2x + 1,$$

should be used to find γ .

It is found that $g'(\gamma) \approx -0.977$.

- (a) Explain what this tells you about the speed of convergence of the sequence of approximations to γ which would be generated by $x_{n+1} = g(x_n)$. [2]

The relaxed iteration

$$x_{n+1} = (1 - \lambda)x_n + \lambda g(x_n) \text{ with } \lambda = 0.506$$

is used to find a sequence of approximations to γ .

The spreadsheet output is shown in the table, together with the values of $f(x_r) = 8 \ln(x_r + 2) - 3x_r + 1$ for $r = 0, 1, 2, 3, 4$.

	E	F	G	
1	r	x_r	$f(x_r)$	
2	0	5	1.567281192	
3	1	5.7930443	0.046719768	
4	2	5.8166845	3.04288E-05	
5	3	5.8166999	-4.07401E-09	
6	4	5.8166999	5.47118E-13	

- (b) State the value of γ as accurately as you can, justifying the precision quoted. [1]
- (c) Explain why the values displayed in cells F5 and F6 are the same as each other, but the values in cells G5 and G6 are different to each other. [2]

The negative root of the equation $8 \ln(x+2) - 3x + 1 = 0$, δ , is close to -1 .

It is found that $g'(\delta) \approx 13.9$.

- (d) Explain why the iterative formula

$$x_{n+1} = g(x_n) \text{ with } x_0 = -1$$

may **not** be used to find δ .

[1]

(e) (i) Use the relaxed iteration

$$x_{n+1} = (1 - \lambda)x_n + \lambda g(x_n) \text{ with } \lambda = 0.506 \text{ and } x_0 = -1$$

to find x_1, x_2, x_3 and x_4 , giving your answers correct to 3 decimal places. [2]

(ii) State what would happen if the iteration in part **(e)(i)** were to be continued. [1]

(f) Use the relaxed iteration

$$x_{n+1} = (1 - \lambda)x_n + \lambda g(x_n) \text{ with } \lambda = -0.078 \text{ and } x_0 = -1$$

to determine the value of δ correct to 7 decimal places. [2]

8

- 8 Following an injury an athlete returns to training in preparation for a regional competition. At the end of each week the athlete's trainer records the time, t seconds, the athlete takes to run 100 metres.

Table 8.1 shows some of the times recorded at the end of week W .

Table 8.1

W	0	2	3
t	18.00	17.68	16.83

The trainer uses the information in the table to construct a polynomial of degree 2 to model these data.

- (a) Use Lagrange's interpolating polynomial of degree 2 to obtain the trainer's model. [3]

Subsequently it is found that when $W = 4$, $t = 15.44$.

- (b) Determine whether the model is a good fit for these values. [1]

The trainer decides to use the three most recent data points, when $W = 2, 3$ and 4 , to construct a new model using Newton's forward difference interpolation formula.

Table 8.2 shows the difference table for these data.

Table 8.2

W	t		
2	17.68		
		-0.85	
3	16.83		-0.54
		-1.39	
4	15.44		

- (c) The trainer obtains the interpolation formula

$$t = -0.27W^2 + BW + C, \text{ where } B \text{ and } C \text{ are constants.}$$

- Determine the value of B .
- Determine the value of C . [3]

- (d) Explain why the new model is a refinement of the model found in part (a). [1]

At the regional competition, the athlete needs to run 100 metres in less than 11 seconds in order to qualify for a national competition. The regional competition takes place when $W = 6$.

- (e) Determine whether, according to the new model, the athlete will qualify for the national competition. [1]
- (f) Explain whether it would be appropriate to use the new model to predict the athlete's times for $W \geq 7$. [1]

END OF QUESTION PAPER

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